AYLANA AKSLANTIRISHLARIDA BURISH SONINING MUNOSIB KASRI HAMDA UNING MAXRAJI HAQIDA TEOREMA

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Annotatsiya: Ushbu ishda burish soni $\rho-$ irratsional bo`lgan yo‘nalishni saqlovchi T aylana gomeomorfizmi qaralgan. T gomeomorfizmning burish sonlari $\rho^{(1)}(T)=\rho_f^{(1)}=[k_1,k_2,k_1,k_2,...,k_1,k_2,...]$, va $\rho^{(2)}(T)=\rho_f^{(2)}=[k_2,k_1,k_2,k_1,...,k_2,k_1,...]$, $k_1,k_2\in N$ bo‘lganda $\frac{p_n}{q_n}$ munosib kasrning maxraji q_n uchun formulalar topilgan.

Kalit so‘zlar: Burish soni, yo‘nalishni saqlovchi aylana gomeomorfizmlari, munosib kasr.

Asosiy qism.

Dinamik sistemalar nazariyasida haqiqiy sonlarni uzluksiz kasrlarga yoyilmasi muhim ahamiyatga ega. Ixtiyoriy $\rho\in(0,1)$ haqiqiy sonni bir qiymatli quyidagi ko'rinishda yoyiladi([1]-[3]):

$$\rho=\cfrac{1}{k_1+\cfrac{1}{k_2+\cfrac{1}{k_3+...\cfrac{1}{k_n+...}}}} \tag{1}$$

Bu yerda $k_1,k_2,...,k_n,...$ - natural sonlar. (1) ifodaga uzluksiz kasr deyiladi va u $\rho=[k_1,k_2,...,k_n,...]$ ko‘rinishda belgilanadi. Ixtiyoriy haqiqiy a sonini $[a]+\{a\}$ ko'rinishida ifodalash mumkin bolgani uchun, $a\in[0,1)$ bo'lgan holni qarash yetarli. $\frac{p_n}{q_n}=[k_1,k_2,...,k_n]$, $n\geq 1$ deb olamiz va bular ρ sonning munosib kasrlari deyiladi, bu yerda p_n va q_n lar barcha $n\geq 1$ lar uchun quyidagi rekurrent munosabatlarni qanoatlantiradi:

$$p_{n+1}=k_{n+1}p_n+p_{n-1},\quad p_0=0, p_1=1,$$
$$q_{n+1}=k_{n+1}q_n+q_{n-1},\quad q_0=1, q_1=a_1.$$

$\left\{\frac{p_{2n-1}}{q_{2n-1}}, n\geq 1\right\}$ ketma-ketlik monoton o'sib, $\left\{\frac{p_{2n}}{q_{2n}}, n\geq 1\right\}$ ketma-ketlik monoton kamayib, ρ songa yaqinlashadi.

Burish soni $\rho-$ irratsional bo‘lgan yo‘nalishni saqlovchi T aylana gomeomorfizmini qaraymiz. ρ ning uzluksiz kasrga yoyilmasi: $\rho=[k_1,k_2,...,k_n,...]$, $\frac{p_n}{q_n}$ orqali ρ sonining $n-$ munosib kasrini belgilaymiz. Ixtiyoriy $x_0\in S^1$ olamiz va uning trayektoriyasini qaraymiz.

$$O(x_0)=\{x_i=T^i x_0,\ i\geq 0\}$$

x_{q_n} va $x_{q_{n+1}}$ nuqtalar x_0 nuqtaning turli tomonlarida yotadi. Aniqrog‘i n toq bo‘lsa x_{q_n} nuqta x_0 nuqtaning chap tomonida n juft bo‘lsa, x_0 nuqtaning o‘ng tomonida yotadi.([1]-[3])

Faraz qilaylik T gomeomorfizmning burish sonlari

$$\rho^{(1)}(T)=\rho_f^{(1)}=[k_1,k_2,k_1,k_2,...,k_1,k_2,...],\qquad \rho^{(2)}(T)=\rho_f^{(2)}=[k_2,k_1,k_2,k_1,...,k_2,k_1,...]$$

bo‘lsin.

Teorema. $\rho_f^{(1)}$ va $\rho_f^{(2)}$ burish sonlarining uzluksiz kasrga yoyilmasi mos holda $\rho_f^{(1)}=[k_1,k_2,k_1,k_2,...,k_1,k_2,...]$ va $\rho_f^{(2)}=[k_2,k_1,k_2,k_1,...,k_2,k_1,...]$ ko‘rinishda bo‘lib, $\frac{p_n}{q_n}$ munosib kasrning maxraji q_n uchun $\rho_f^{(1)}=[k_1,k_2,k_1,k_2,...,k_1,k_2,...]$ bo‘lganda $q_0=1, q_1=k_1,\ n\geq 1$ va $\rho_f^{(2)}=[k_2,k_1,k_2,k_1,...,k_2,k_1,...]$ bo‘lganda $q_0=1, q_1=k_2,\ n\geq 1$ boshlang‘ich shartlarda mos holda

$$q_n=\frac{k_1\rho_1+1}{\rho_1^2+1}\rho_1^n+(-1)^n\frac{\rho_1^2-k_1\rho_1}{\rho_1^2+1}\rho_1^{-n},\quad q_n=\frac{k_2\rho_2+1}{\rho_2^2+1}\rho_2^n+(-1)^n\frac{\rho_2^2-k_2\rho_2}{\rho_2^2+1}\rho_2^{-n},\quad n\geq 0$$

munosabat o‘rinli.

Xulosa. Irratsional burish sonli yo‘nalishni saqlovchi aylana akslantirishlarida juda muhim o‘rin tutadigan Irratsional burish soni munosib kasrining maxraji haqida teoremani keltirdik.

Foydalanilgan adabiyotlar ro‘yxati:

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