Fan–texnika taraqqiyoti jamiyatning moddiy va ma’naviy ehtiyojlarini har tomonlama qondirish uchun keng imkoniyatlar ochib berayotgan bo’lsa, ikkinchi tomondan tabiiy resurslardan tobora intensiv foydalanishni taqozo qilmoqda. Bu jarayon tevarak–atrofdagi muhitga, ekologik muvozanatga salbiy ta’sir ko’rsatmoqda. Shuning uchun ham respublikamiz taraqqiyotining hozirgi bosqichida tabiiy resurslardan oqilona foydalanish, o’rmon, toza havo, hayvonot dunyosi, yer–suv, ekologik toza mahsulotlar ishlab chiqarish to’g’risida g’amxo’rlik qilish har birimizning muqaddas burchi bo’lmog’i kerak.

Foydalanilgan adabiyotlar ro’yhati:

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AYLANISH SIRTINING YUZINI HISOBLASH USULLARI

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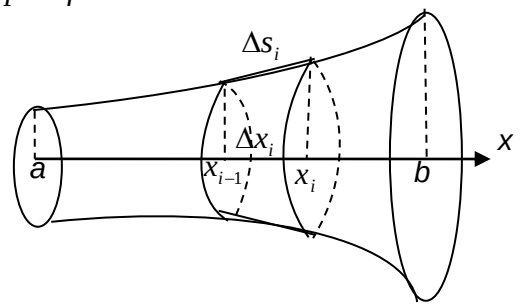
Annotatsiya. Ushbu maqolada aylanish sirtlarining yuzini hisoblash usullari keltirilgan va unga doir misollar ham berilgan.

Kalit so‘zlar. Aylanish sirtlari, differensiallanuvchi funksiya, O*x* o‘qi atrofida aylantirish natijasida hosil qilingan sirt, Gorizontal aylanma sirt yuzasini hisoblash, Vertikal aylanma sirt yuzasini hisoblash.

Aytaylik, [*a;b*] kesmada manfiy bo‘lmagan uzluksiz differensiallanuvchi *y=f(x)* funksiya grafigini O*x* o‘qi atrofida aylantirish natijasida sirt hosil qilingan bo‘lsin (1-rasm). Shu aylanish sirtining yuzini hisoblashda quyidagi formuladan foydalanamiz:

$$S = 2\pi \int_a^b f(x) \cdot \sqrt{1 + [f'(x)]^2}\, dx \qquad (1)$$

Bu *aylanish sirtining yuzini topish formulasidir*.



1-rasm

Gorizontal aylanma sirt yuzasini hisoblash:

> SurfaceOfRevolution(f(x), x=a..b, output=integral);

$$\int_a^b 2\pi |f(x)| \sqrt{\left(\frac{d}{dx} f(x)\right)^2 + 1}\, dx$$

Vertikal aylanma sirt yuzasini hisoblash:

> SurfaceOfRevolution(f(x), x=a..b, output=integral, axis=vertical);

$$\int_a^b 2\pi |x| \sqrt{\left(\frac{d}{dx} f(x)\right)^2 + 1}\, dx$$

1 misol. $y = \sqrt{x}$ funksiya grafigining $x \in [1;2]$ kesmaga mos qismini Ox o‘qi atrofida aylantirishdan hosil bo‘lgan sirtning yuzini hisoblang.

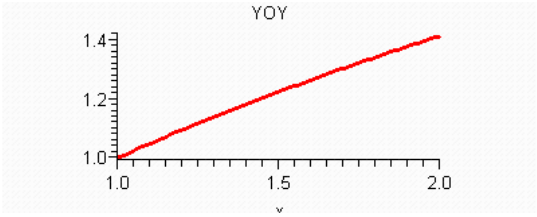
Yechish. $f(x) = \sqrt{x}$, $f'(x) = \frac{1}{2\sqrt{x}}$. Bularni (1) ga qo‘yamiz.

$$S = 2\pi \int_1^2 \sqrt{x} \cdot \sqrt{1 + \left(\frac{1}{2\sqrt{x}}\right)^2}\, dx = 2\pi \int_1^2 \sqrt{x + \frac{1}{4}}\, dx =$$

$$= 2\pi \cdot \frac{2}{3}\left(\sqrt{x + \frac{1}{4}}\right)^3 \Big|_1^2 = \frac{4\pi}{3} \cdot \left(\left(\sqrt{2 + \frac{1}{4}}\right)^3 - \left(\sqrt{1 + \frac{1}{4}}\right)^3\right) =$$

$$= \frac{4\pi}{3}\left(\frac{27}{8} - \frac{5\sqrt{5}}{8}\right) = \frac{27 - 5\sqrt{5}}{6}\pi \approx 8{,}283.$$

>restart; with(plots): with(Student[Calculus1]):
>f:=x->sqrt(x):
>plot(f(x),x=1..2,color=red,style=line, thickness=2, title=`YOY`);



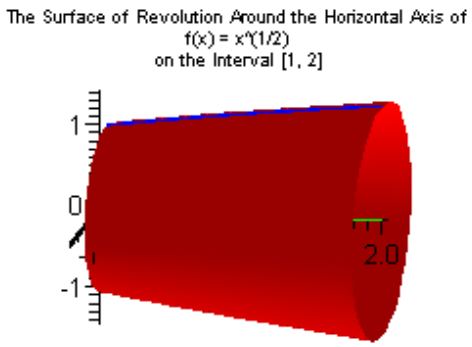
$$\int_1^2 \pi \sqrt{x} \sqrt{\frac{1 + 4x}{x}}\, dx$$

> SurfaceOfRevolution(f(x), x=1..2, output=integral);

> SurfaceOfRevolution(f(x), x=1..2); $\frac{5}{6}\sqrt{5}\pi + \frac{9}{2}\pi$

> evalf(%); 283154665

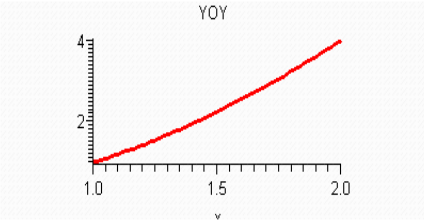
> SurfaceOfRevolution(f(x), x=1..2, output=plot);



2-misol. $y = x^2$ funksiya grafigining $x \in [1;2]$ kesmaga mos qismini:

Oy o‘qi atrofida aylantirishdan hosil bo‘lgan sirtning yuzini hisoblang.

```
>restart; with(plots): with(Student[Calculus1]):
>f:=x->x^2:
>plot(f(x),x=1..2,color=red,style=line, thickness=2, title=`YOY`);
```




```
> SurfaceOfRevolution(f(x), x=1..2, output=integral, axis=vertical);
```

$$\int_1^2 2\pi x\sqrt{4x^2+1}\,dx$$

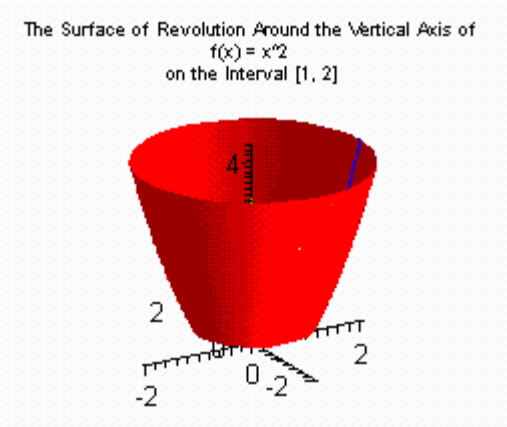
```
> SurfaceOfRevolution(f(x),x=1..2,axis=vertical);
```

$$\frac{5}{6}\sqrt{5}\pi+\frac{17}{6}\sqrt{17}\pi$$

```
> evalf(%);
```

30.84648972

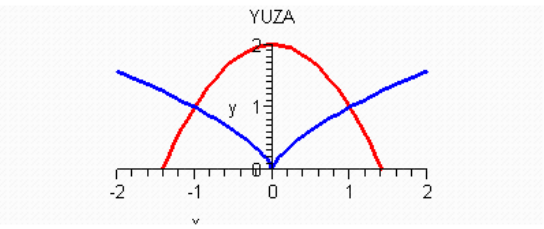
```
> SurfaceOfRevolution(f(x), x=1..2, output=plot,axis=vertical);
```



3-misol. $y = 2 - x^2$, $y^3 = x^2$ chiziqlar bilan chegaralangan yassi figyrani Oy *va* Ox o’qiga atrofida aylanishsan hosil bo’lgan sirtning yuzasini hisoblang.

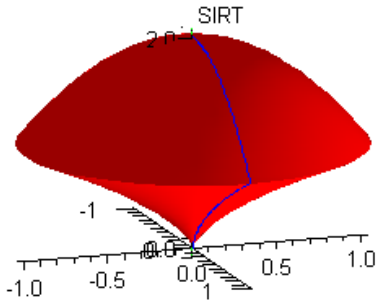
1) ***Berilgan*** *chiziqlar bilan chegaralangan yassi figyra:*

```
> restart; with(plots):    with(Student[Calculus1]):
> f2:=x->2-x^2: f1:=x->(x^2)^(1/3):
> plot({f2(x),f1(x)},x=-2..2, y=0..2,color=[red,blue],
style=line,thickness=2,title=`YUZA`);
```



2) *figyrani Oy o’qiga atrofida aylanishsan hosil bo’lgan sirtni qurish:*

> S1:=SurfaceOfRevolution(f2(x), x=0..1, output=plot, axis=vertical):
S2:=SurfaceOfRevolution(f1(x), x=0..1, output=plot, axis=vertical):
display({S2,S1}, axes=boxed, title=`SIRT`);



3) *figyrani Oy o'qiga atrofida aylanishsan hosil bo'lgan sirtning yuzasini hisoblash*
> SurfaceOfRevolution(f2(x), x=0..1, output=integral,
axis=vertical)+SurfaceOfRevolution(f1(x), x=0..1, output= integral,
axis=vertical);

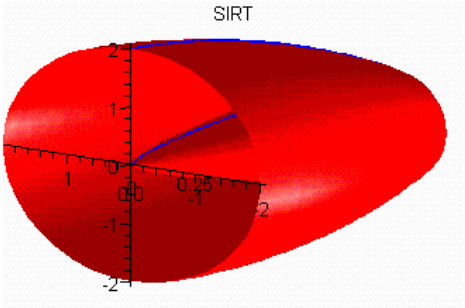
$$\int_0^1 2\,\pi\,x\sqrt{4x^2+1}\,dx+\int_0^1 \frac{2}{3}\,\pi\,x\sqrt{\frac{4+9x^{(2/3)}}{x^{(2/3)}}}\,dx$$

> SurfaceOfRevolution(f2(x), x=0..1,axis=vertical)+
SurfaceOfRevolution(f1(x), x=0..1,axis=vertical);

$$\frac{1}{6}\,\pi\,(5\sqrt{5}-1)+\frac{28}{81}\,\pi\sqrt{13}-\frac{8}{243}\,\pi\ln(2)+\frac{8}{243}\,\pi\ln(3+\sqrt{13})$$

> evalf(%); 369550289

4) *figurani Ox o'qiga atrofida aylanishsan hosil bo'lgan sirtni qurish.*
> S1:=SurfaceOfRevolution(f2(x), x=0..1, output=plot):
S2:=SurfaceOfRevolution(f1(x), x=0..1, output=plot):
display({S2,S1}, axes=boxed, title=`SIRT`);



5) *figurani Ox o'qiga atrofida aylanishsan hosil bo'lgan sirtning yuzasini hisoblash.*
> SurfaceOfRevolution(f2(x), x=0..1, output=integral)+
SurfaceOfRevolution(f1(x), x=0..1, output=integral);

$$\int_0^1 K\,2\,p\,(K\,2\,C\,x^2)\,\sqrt{4x^2\,C\,1}\,dx$$

$$C\int_0^1 \frac{2}{3}\,p\,x^{(2/3)}\sqrt{\frac{4\,C\,9x^{(2/3)}}{x^{(2/3)}}}\,dx$$

```
> SurfaceOfRevolution(f2(x), x=0..1)+
SurfaceOfRevolution(f1(x), x=0..1);
```

$$\frac{33}{16}\,p\,\ln(2)\,C\,\frac{23}{16}\,p\,\sqrt{5}\,C\,\frac{33}{32}\,p\,\ln\!\left(\frac{1}{2}\right)\,C\,\frac{1}{4}\,\sqrt{5}\,1C\,\frac{128}{1215}\,p\,C\,\frac{494}{1215}\sqrt{13}\,p$$

```
> evalf(%);     171163206
```

3-misol. $x^{2/3}+y^{2/3}=a^{2/3}$ astroida-chizig'i bilan chegaralangan egri chiziqli yassi yuzani O*x* o'qi atrofida aylanishdan hosil bo'lgan sirt yuzasini hisoblash.

Yechish: O*x* o'qi atrofida aylanishdan hosil bo'lgan sirt yuzasini hisoblash:

$$S_x = 2\pi\int_\alpha^\beta y\sqrt{(x')^2+(y')^2}\,dt$$

formulasiga asosida, astroidaning 101arametric tenglamasi $x=a\cos^3(t)$, $y=a\sin^3(t)$ $(\alpha<t<\beta)$ga asosan

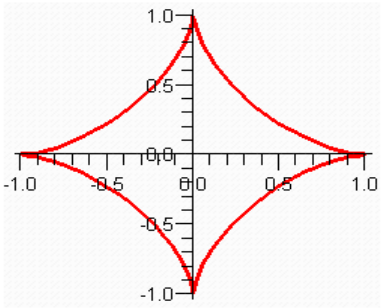
$$S_x = 2\cdot 2\pi\int_\alpha^\beta y\sqrt{(x')^2+(y')^2}\,dt =$$

$$S_x = 2\cdot 2\pi\int_\alpha^\beta y\sqrt{(x')^2+(y')^2}\,dt = 4\pi\int_0^{\pi/2} a\sin^3 t\sqrt{9a^2\cos^4 t\sin^2 t+9a^2\sin^4 t\,\cos^2 t}\,dt =$$

$$= 4\pi a\int_0^{\pi/2}\sin^2 t\,3a\sin t\cos t\,dt = 12\pi a^2\int_0^{\pi/2}\sin^4 t\,d(\sin t) = \frac{12\pi a^2}{5}\sin^5 t\Big|_0^{\pi/2} = \frac{12\pi a^2}{5}$$

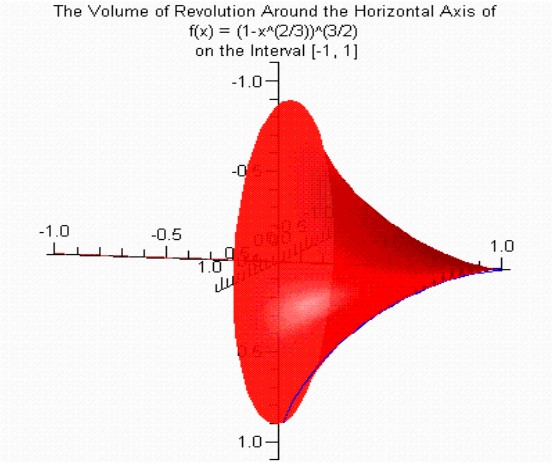
1) *astroida grafigini quramiz*:

```
> with(plots):
> plot([1*cos(t)^3,1*sin(t)^3, t=0..2*Pi],thickness=2);
```



2) *astroidani* [*0,π/2*] *kesmasiga mos qimining Ox o'qiatrofida aylanishdan hosil bo'lgan sirt grafigi.*

```
> restart; with(plots):    with(Student[Calculus1]):
> VolumeOfRevolution(sqrt(1^(2/3)-x^(2/3))^3, x=0..1, output=plot);
```



3) *astroida-chizig'I bilan chegaralangan figurani* [0,π/2] *kesmasiga mos qismini Ox o'qiatrofida aylanishdan hosil bo'lgan sirt yuzasini hisoblab, uni ikkiga ko'paytirib, to'la sirtni topamiz.*

```
> with(Student:-Calculus1):
> x:=t->a*cos(t)^3; y:=t->a*sin(t)^3
```

$$x := t \to a\cos(t)^3 \qquad y := t \to a\sin(t)^3$$

```
> SS1:=(2*Int(2*Pi*y(t)*sqrt(diff(x(t),t)^2+diff(y(t),t)^2), t=0..Pi/2));
```

$$SS1 = 2\int_0^{\frac{1}{2}\pi} 6\pi a\sin(t)^3 (a^2\cos(t)^4\sin(t)^2 + a^2\sin(t)^4\cos(t)^2)^{\frac{1}{2}}\,dt$$

```
> value(%);
```

$$\frac{12}{5}\pi a^2\,\mathrm{csgn}(a)$$

```
> evalf(%);
```

$$7.539822370a^2\,\mathrm{csgn}(a)$$

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