

Method of Controlling Reinforcement Forces to Win in Combat Conflict Situations

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Abstract: In this article, problems related to the control of reinforcements coming in opposite directions, based on the Lanchester's extended quadratic law. The rate of receipt of reinforcements is taken as a piecewise constant function. A successful strategy for managing auxiliary forces arriving to the conflicting parties is described. On the basis of the model of combat operations, software for the management of auxiliary forces was created. The results of the battle between Japan and America on the island of Iwo Jima and other conflict situations were also simulated.

1 INTRODUCTION

In recent years, a number of results have been achieved in the research of command decision-making in combat operations based on Lanchester's quadratic law. Today, on the basis of Lanchester's quadratic law, the issues of optimal control and distribution of auxiliary power resources leading to success are deeply studied. It is worth noting that the development of decision-making strategies and optimal strategies for winning according to strengths has become a focus of research. This article also proposes methods for controlling auxiliary forces to win in conflict situations based on Lanchester's extended quadratic law.

The following works can be cited regarding the description and simulation of combat actions based on Lanchester's quadratic law. For example, in the monograph "Lanchester-type war models" by J.G.Taylor, the combat actions of opposing forces are widely covered on the basis of differential equations and analyzed in depth on the basis of examples [1]. In his dissertation, Gerardo Minguela Castro empirically evaluated the battle of Crete, the battles of Iwo Jima and Kursk, as well as three influential battles of the Second World War [2].

In [3], J.G.Taylor and G.G.Brown developed canonical methods for solving Lanchester-type

equations with variable coefficients for modern warfare.

In [4], C.Sha and J.Zeng show that which side has the upper hand in the battle, according to the firepower of weapons, the number of soldiers and combat equipment on which side has more chances to win, Lanchester's equations and analyzed based on the theory of differential games.

Now, let's consider the situation of combat conflict between two opposing sides based on Lanchester's quadratic law.

In this article, we present the methods, algorithm and results of the control of auxiliary forces that come to the parties to win in combat conflict situations.

2 Description of the Combat Model with Reinforcement forces

In a conflict situation between two opposing sides with the same type of forces, we consider the model of combat operations, taking into account the incoming reinforcement forces. For example, if reinforcement forces are added to sides A and B at speeds $u(t)$ and $v(t)$, respectively, then the rate of change of their forces is as follows:

$$\begin{cases} \dot{x}_1(t) = -a_1 x_1(t) - b_1 x_2(t) + u(t) \\ \dot{x}_2(t) = -a_2 x_2(t) - b_2 x_1(t) + v(t) \end{cases} \quad (1)$$

Where, $x_1(t)$ and $x_2(t)$ are the number of forces of the opposing sides at the moment of time t ;
 $a_{1,2}$ - coefficient of non-combat losses of the parties;
 $b_{1,2}$ - coefficient of losses due to hostilities of the parties; $x_1(0) = x_{10}$ and $x_2(0) = x_{20}$ - the number of initial forces of the parties; $u(t)$ and $v(t)$ are the speed of arrival of reinforcement forces, i.e. control of the parties.

Next, we consider the speed of arrival of auxiliary forces in military conflicts as parameters of control of the parties.

We use the following assumptions and definitions to solve the above problem. It should be noted that the auxiliary forces of each side are made up of a limited number of soldiers and weapons reserves.

Assumption 1. We assume that the functions $u(t)$ and $v(t)$ are, piecewise-constant functions in the interval $[0, T]$.

Definition 1. The function $f \in B[t, T]$ is called piecewise-continuous if, $[t, T]$ is divided by $t_1, t_2, \dots, t_{n-1}$ points such that in each (t_{k-1}, t_k) interval the function f is continuous and has one-sided limits $\lim_{t \rightarrow t_{k-1}+0} f(t)$ and $\lim_{t \rightarrow t_k-0} f(t)$. The value of the function f at t_k points is not important. Where, $t = 0 < t_1 < t_2 < \dots < t_{n-1} < t_n = T$.

Definition 2. The function f is called piecewise-constant in the section $[t, T]$, if it is piecewise-continuous in every section (t_{k-1}, t_k) and invariant in every interval of continuity.

Case 1.

Assumption 2. Let X_0 and Y_0 denote the total number of auxiliary forces coming to the opposite sides, and let them satisfy the following conditions:

$$u(t) = \begin{cases} d, & \text{if } 0 \leq t \leq \frac{X_0}{d} \\ 0, & \text{if } t > \frac{X_0}{d} \end{cases} \quad \text{or} \quad u(t) = \begin{cases} d, & \text{if } 0 \leq t \leq T_u \\ 0, & \text{if } t > T_u \end{cases} \quad (2)$$

$$v(t) = \begin{cases} k, & \text{if } 0 \leq t \leq \frac{Y_0}{k} \\ 0, & \text{if } t > \frac{Y_0}{k} \end{cases} \quad \text{or} \quad v(t) = \begin{cases} k, & \text{if } 0 \leq t \leq T_v \\ 0, & \text{if } t > T_v \end{cases}$$

Assumption 3. We assume that the number of auxiliary forces is added to the opposite sides at a constant speed until it reaches the total number of reserves:

$$\int_0^{T_u} u(t) dt = X_0 \quad \text{and} \quad \int_0^{T_v} v(t) dt = Y_0 \quad (3)$$

where T_u, T_v are the total deployment times of the auxiliary forces for the respective sides.

There are two possible situations to consider here.

Case 1a. The total time to spend the auxiliary forces can be longer than the time to finish the battle. In this case, all reserve forces are not fully used ($T_u \geq T$ as well as $T_v \geq T$).

Case 1b. The total time spent by the auxiliary forces can be less than or equal to the end of the battle. In this case, all reserve forces are fully used ($T_u \leq T$ as well as $T_v \leq T$).

Definition 2. The winning strategy of party X is the control $u(t)$ for which $T_u \geq T$ (or $T_u \leq T$) holds for inequality

$$x_1(T) \geq M, \quad x_2(T) = 0. \quad (4)$$

Case 2.

Auxiliary forces join the conflicting parties either at random times or continuously throughout the battle as shown in Figure 1 below.

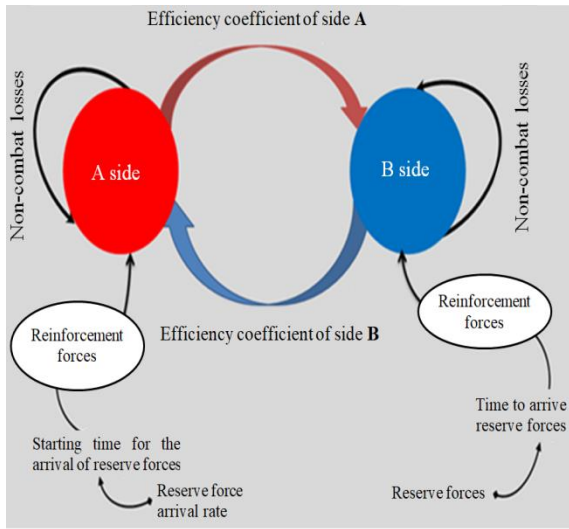


Figure 1. Schematic of Lanchester's Augmented Quadratic Model Including Auxiliary Forces and Non-Combat Losses

3 RESULTS AND DISCUSSIONS

Now, let's see how Lanchester's square law applies to combat. Based on the above assumptions and conditions, we implement the model (1) in practical examples.

Problem 1. As an example, let's take the 36-day battle between America and Japan on the island of Iwo Jima, which lasted from February 19 to March 26, 1945, [5, 6, 7]. In this case, we consider side A as the Americans, and side B as the Japanese. For the simulation of the Battle of Iwo Jima, the parameters in model (1) are equal to $a_1 = a_2 = v(t) = 0$. In [5], J.H.Engel estimated the coefficients of efficiency (the rate of losses of the parties due to combat action) for the Americans and the Japanese as follows.

Table 1. Efficiency coefficients of Americans and Japanese

Efficiency coefficient of the Japanese	0,05440
Efficiency coefficient of the Americans	0,01060

These coefficients of combat effectiveness correspond to values $b_1 = 0.0106$ and $b_2 = 0.0544$ for model (1). Also, in this battle, the American side received reinforcements twice, and the Japanese did not. Below is the amount of reinforcements that have arrived on the American side.

$$u(t) = \begin{cases} 0, & 0 < t \leq 1 \\ 0, & 1 < t \leq 2 \\ 6000, & 2 < t \leq 3 \\ 0, & 3 < t \leq 4 \\ 0, & 4 < t \leq 5 \\ 13000, & 5 < t \leq 6 \\ 0, & 6 < t \leq 36 \end{cases} \quad (5)$$

During this battle, the Americans landed two reinforcements: 6,000 troops on the third day of the battle and 13,000 on the sixth day. So, according to definition 2, the management chosen by the Americans is a winning strategy. Below is the simulation result of Problem 1.

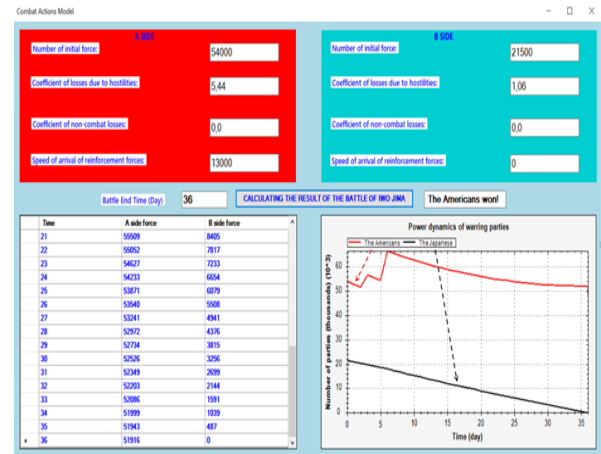


Figure 2. Simulation result of the Battle of Iwo Jima

In all, about 28,000 American soldiers were killed in this battle, including about 8,000 killed, while the Japanese lost more than 20,000 soldiers killed and about 1,000 captured. Also, the cost of taking the island of Iwo Jima was more than the Americans expected [6].

Problem 2. A battle ensued between two sides, A and B, with 1,500 soldiers each. The efficiency coefficients of the parties are 0.03, and the coefficients of non-combat losses are 0.003. Also, each side has 600 reserve or auxiliary forces. The two opposing sides manage and distribute their auxiliary forces as follows. Let these auxiliary forces reinforce both sides from the 15th day of the battle. If side A receives 300 soldiers on the 15th day of the battle, then the remaining 300 soldiers join on the 75th day of the battle, and side B receives 10 soldiers continuously from the 15th day of the battle until the 75th day. , then how many days will the battle last and which side will win?

Numerical solution. The mathematical expression of the available reserve forces on side A can be written as:

$$\int_0^1 u_1(t)dt + \int_1^2 u_2(t)dt = [u_1(t) = u_2(t)] = \int_0^2 u_1(t)dt = X_0$$

here $u(t)$ is the speed of arrival of auxiliary forces to A, i.e. equal to 300.

For side B:

$$\int_{20}^{120} v(t)dt = Y_0$$

here, $v(t)$ is the speed of arrival of auxiliary forces to side B is equal to 10. The meaning of this problem is reflected in Figure 1 above. Considering the speed of arrival of auxiliary forces to opposing parties as a control function, the control algorithm is implemented as follows:

Table 2. Algorithm of control functions of parties

<pre> int ControlU(int t) { int u; if (t == 15) u = 300; else if (t == 75) u = 300; else u = 0; return u; } </pre>	<pre> int ControlV(int t) { int v; if (t >= 15 && t <= 75) v = 10; else v = 0; return v; } </pre>
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Now, based on the initial conditions $x_1(0) = 1500$, $x_2(0) = 1500$ and combat and non-combat losses $a_{1,2} = 0.003$, $b_{1,2} = 0.03$, we find the numerical and graphical solutions of problem 2 using the modified Euler method.

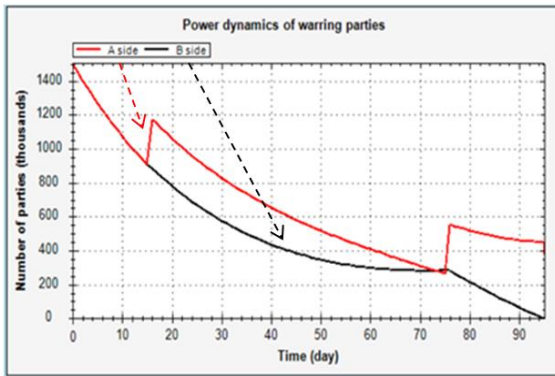


Figure 3. Dynamics of changing the forces of warring parties

This battle will last 95 days based on the initial conditions above, resulting in Side A winning and 448 soldiers surviving. According to Definition 2, the

strategy chosen by Party A to control the auxiliary forces is the winning strategy.

Of course, the situation may change if one or more parameters change. For example, if side B continuously reinforces 10 soldiers from day 5 of the battle until day 65, then side B wins in 100 days and 350 soldiers survive. In this case, the strategy chosen by B to control the auxiliary forces is the winning strategy.

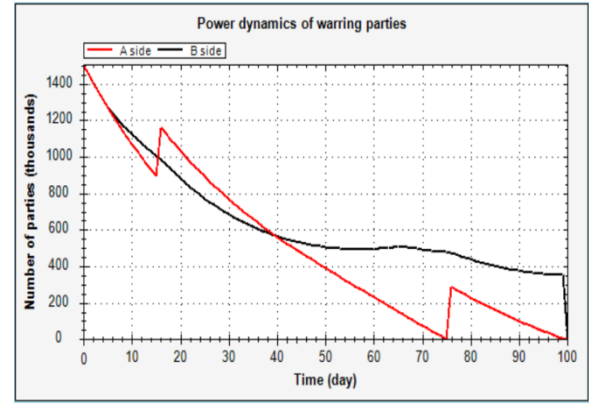


Figure 4. Dynamics of changing the forces of warring parties

Figure 4 above shows the dynamics of changes in the power of parties A and B, where it can be seen that the power of each party decreases according to the exponential law. Also, the arrival of auxiliary forces will increase the strength of the sides, and as a result, the strength of the B side will suffer serious losses from a certain point in time, while the strength of the A side will rapidly decrease and eventually disappear completely.

4 CONCLUSIONS

In this work, two cases of introducing auxiliary forces into the model of combat operations and controlling them were seen. Also provided is a scheme of the combat operations model written with auxiliary forces. The modified Euler method and programming technologies were used to find the numerical solutions of the model (1).

The battle of Iwo Jima was simulated based on J.H.Engel [5]. Based on the model of combat operations, an algorithm and software for the construction of control functions for the distribution of auxiliary forces of opposing parties were created.

Also, the software created can calculate the amount of forces left on each side, how long the conflict will last, which side will win, combat and non-combat loss ratios, taking into account the arrival

of reinforcements in combat situations. allows you to determine how it affects the outcome of the battle and simulate various military conflicts.

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