// National Database of Legislation of the Republic of Uzbekistan – https://lex.uz/docs/4208064.

7. Resolution No.366 of the Cabinet of Ministers of the Republic of Uzbekistan dated October 27, 2016 "On the responsibility of heads of ministries and departments, business associations, Chairman of the Council of Ministers of the Republic of Karakalpakstan, khokimiyats of regions, cities and districts for unconditional compliance with the law" // National Database of Legislation of the Republic of Uzbekistan – https://lex.uz/docs/3058254.

8. Code of the Republic of Uzbekistan on Administrative Responsibility // National Database of Legislation of the Republic of Uzbekistan – https://lex.uz/docs/97664.

9. Criminal Code of the Republic of Uzbekistan – https://lex.uz/docs/111453.

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STEFFENSEN (EITKEN-STEFFENSEN) METHOD FOR SOLVING NONLINEAR EQUATIONS

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Abstract: Solving nonlinear equations is more complicated and is a perfectly unresolved problem in computational mathematics. This iterative algorithm is called the Steffensen method in numerical methods. The Steffensen method has a quadratic approximation. This method requires calculating the value of the function twice in each iteration, in which case the Steffensen method is less efficient than the cutters method.

Keywords: Steffensen method, iterative algorithm, approximation, Eitken, Newton method, sequence, limit, linear approximation.

To increase the approximation speed of the test method

$x_n + 1 = \varphi(x_n)$ n = 0,1,2, … (1)

In the expression $f'(x_n)$ Abbreviation of words that bring the harvest closer:

$f'(x_n) \approx \frac{f(x_{n+1}) - f(x_n)}{x_{n+1} - x_n}$ (2)

If (1) is a left-handed approximation, then (2) is a right-handed approximation. (2) shows that in which it has not yet been determined x_n+1 To calculate the presence of an unknown limit, we use a simple iteration (1.2):

$x_{n+1} = g(x_n) = x_n + f(x_n).$

As a result, we have the following approximation:

$$f'(\mathrm{x_n}) \approx \frac{f(\mathrm{x_n}+f(\mathrm{x_n}))-f(\mathrm{x_n})}{f(\mathrm{x_n})}.$$

Using this expression in Newton's method, we get a new iterative algorithm:

$$\mathrm{x_{n+1}} = \mathrm{x_n} - \frac{f(\mathrm{x_n})}{f(\mathrm{x_n}+f(\mathrm{x_n}))-f(\mathrm{x_n})} f(\mathrm{x_n}). \ (3)$$

This iterative algorithm is called the Steffensen method in numerical methods. The Steffensen method has a quadratic approximation, but here in addition $f(\mathrm{x_n} + f(\mathrm{x_n}))$ a high approximation speed is achieved by calculating the value of the expression. .The iterative algorithm above (3) can also be derived from the method of accelerating the convergence of linear convergent sequences proposed by Eitken.

To do this, consider the following sequence:

$$\mathrm{z_n} = z + Cq^n \ (4)$$

This is a sequence $|q| < 1$ to z limit approached. Find the limit value of z using simple reflections $\{\mathrm{z_n}\}$ three in a row $\mathrm{z_{n-1}}, \mathrm{z_n}$ *and* $\mathrm{z_{n+1}}$ in series $\frac{\mathrm{z_n}-z}{\mathrm{z_{n-1}}-z} = q$ and $\frac{\mathrm{z_{n+1}}-z}{\mathrm{z_n}-z} = q$ this from two equations $(\mathrm{z_{n+1}} - z)(\mathrm{z_{n-1}} - z) = ((\mathrm{z_n} - z)^2$ equality. This leads to the following expression for z:

$$\mathrm{z} = \frac{\mathrm{z_{n+1}z_{n-1}} - \mathrm{z_n^2}}{\mathrm{z_{n+1}} - 2\mathrm{z_n} - \mathrm{z_{n-1}}}.$$

Based on this result, let us consider the following Eitken proposition to replace the $\{\mathrm{z_n}\}$ sequence with another sequence:

$$\xi_{\mathrm{n+1}} = \frac{\mathrm{z_{n+1}z_{n-1}} - \mathrm{z_n^2}}{\mathrm{z_{n+1}} - 2\mathrm{z_n} + \mathrm{z_{n-1}}}. \ (5)$$

If this substitution is in any sequence in the form (4)

if we use, then at any value of n $\xi_{\mathrm{n}} = z - \lim_{n\to\infty} z^n$ equality is appropriate. If the approximation type of the sequence $\{\mathrm{z_n}\}$ is close to (4), then substitution (5) gives z a new sequence that approximates z faster than the original (even if it does not give its limit at an arbitrary value of n).

1-for example. This

$$x^3 - x^2 - 8x + 12 = 0$$

approximate the double root of the equation $\mathrm{x_r} = 2$ using Newton's method and the Eitken accelerator.

Solution. The results of the calculations are presented in the table below with the elements of the corresponding sequences (see columns three and four).

n	x_n	$\lvert x_n - x_r \rvert / \lvert x_{n-1} - x_r \rvert$	ξ_n	$\lvert \xi_n - x_r \rvert / \lvert \xi_{n-1} - x_r \rvert$
0	0.5	-	-	-
1	1.454545	0.363636	-	-
2	1.745059	0.467391	1.872159	-
3	1.876049	0.486197	1.983607	0.128232
4	1.938822	0.493563	1.996588	0.208141
5	1.969602	0.496884	1.999213	0.230676
6	1.984847	0.498466	1.999811	0.240656
7	1.992435	0.499239	1.999954	0.245400
8	1.996221	0.499621	1.999988	0.247717
9	1.998111	0.499811	1.999997	0.248863
10	1.999056	0.499905	1.999999	0.249432
11	1.999528	0.499953	2.000000	0.249717
12	1.999764	0.499976	2.000000	0.249856
13	1.999882	0.499988	2.000000	0.249948
14	1.999941	0.499994	2.000000	0.250448

In the third column of this table, the approximation velocity is assumed to be a = 1, and the values of the variable C in equation (4) for each iteration are given. The results in the table show that C changes very little during the iterative process and is very close to C = 0.5. As a result, the hypothesis that the rate of approach of the Newtonian method to the multiple root is linear is proved.

Using the linear approximation $\{x_n\}$ sequence (5) to the accelerating formula, we obtain the values of the s in the fourth column of the table ξ_n. By comparing the values in the second and fourth columns of the table, we make sure that we have reached the approximation speed. Indeed, the result obtained in the fourteenth iteration of the Newtonian method can be seen by applying the Newtonian method and the Eitken accelerator q to the same result in its seventh iteration. The results in the fifth column of this table show that such an effective result was achieved not by increasing the convergence velocity index a, but by reducing the variable C to 0.25. First of all iterative for this $x_{n+1} = g(x_n)$

let's spread the right side of the formula to Taylor's row, that is

$g(x_n) = g(x_r + (x_n - x_r)) = x_r + g'(x_r)(x_n - x_r) + O((x_n - x_r)^2)$,

According to this

$x_{n+1} - x_r = g'(x_r)(x_n - x_r) + O((x_n - x_r)^2)$,

And so, $e_n = x_n - x_r$ The following approximate equation can be written for each iteration with square accuracy:

$x_{n+1} - x_r = g'(x_r)(x_n - x_r)$.

Here the sequence $\{x_n\}$ can be expressed by the following formula:

$x_n \approx x_r + [g'(x_r)]^n(x_n - x_r)$

The approximation type of this sequence is the same as that of the sequence (4). This means that the approximation sequence to the root in simple iteration is suitable for applying the approximation acceleration procedure.

In order to ensure that each improvement value calculated using the approximation acceleration procedure is taken into account in subsequent calculations, it must be taken into account immediately. This is done at each step of the iteration as follows: Suppose that the calculations were performed until the value of x_n was calculated; using it we calculate two auxiliary values $x_n^{(1)} = g(x_n)$ and

$x_n^{(2)} = g(g(x_n))$. We apply the accelerator formula (5) to the three values x, $x_n^{(1)}$, and $x_n^{(2)}$, and the result is the following

Assume that $x_n + 1$ approximation:

$$x_n + 1 = \frac{x_n\, g\left(g\,(x_n)\right) - g^2(x_n)}{g\left(g\,(x_n)\right) - 2\,g\,(x_n) + x_n}$$

It appears that Equation (3) is one of the forms of writing the Steffensen iterative formula.

Example 2. The formula (6) is given by this

$$x^3 - x^2 - 8x + 12 = 0$$

Use to find the double root of the equation.

Solution. To do this,

$g(x) = x - \frac{f(x)}{f'(x)}$

corresponding to the Newtonian iteration

from the calculations according to formula (6)

{0.5; 1,87215909; 1,99916211; 1,99999996; 2,00000000}

we create a sequence. Comparing these values with the values of ξ_n in the fourth column of the table above, we can see that the efficiency is increased by adding the accelerator to the algorithm obtained from the line, rather than to the sequence.

The important conclusions of this work are as follows:

1. Solving nonlinear equations is more complicated and is a perfectly unresolved problem in computational mathematics;

2. The initial problem of solving nonlinear equations is the study of the existence, number and interval of solutions of nonlinear equations, which are explained by solving specific examples;

3. The problem of finding the separated root of a nonlinear equation is described in several approximate ways, explained by the solutions of concrete examples;

4. Approximate methods of finding the roots of a nonlinear equation have been studied from simple to complex and with their special cases, which has made it possible to shed more light on the subject;

Thus, the problem of solving nonlinear equations depends on the type of practical problem, the choice of the correct approximate method and the initial condition, the effective use of these methods.

References:

1. Абдухамидов А. У., Худойназаров С. Ҳисоблаш усулларидан амалиёт ва лаборатория машғулотлари. – Тошкент: Ўқитувчи, 1995. – 240 б.

2. Бахвалов Н. С., Жидков Н. П., Кобельков Г. М. Численные методы. – М.: Изд-во Бином. Лаборатория знаний, 2011. – 640 с.

3. Бахвалов Н. С., Корнев А. А., Чижонков Е. В. Численные методы. Решения задач и упражнения. – М.: Изд-во Дрофа, 2009. – 400 с.

4.Бахвалов Н. С., Лапин А. В., Чижонков Е. В. Численные методы в задачах и упражнениях. – М.: Изд-во Бином. Лаборатория знаний, 2010. – 240 с.

5. Вержбицкий В. М. Основы численных методов. – М.: Высшая школа, 2009. – 848с.

6. Воробьева Г.К., Данилова А.Н. Практикум по вычислител ь-ной математике. – М: Высшая школа, 1990.

7.Демидович Б.П., Марон И.А. Основы вычислительной математики. – М.: Наука, 1966.

8. Дьяконов В.П. Maple 9.5/10 в математике, физике иобразовании. – М.: СОЛОН-Пресс, 2006. – 720 с.

9. Жидков В.Н. Вычислительная математика. – М, Академия, 2010. – 208 с.

10. Исраилов М.И. Ҳисоблаш усуллари. 1 - қисм. – Тошкент: Ўқитувчи, 2003. – 430 б.

11. Калиткин Н.Н. Численные методы. – С.Пб.: Изд-во БХВ-Петербург, 2011. – 592 с.

12. Калиткин Н.Н., Альшина Е.А. Численные методы: в 2 кн. Кн. 1. Численный анализ. – М.: Издательский центр «Академия», 2013– 304 с.

13. Копченова Н.В., Марон И. А. Вычислителная математика в примерах и задачах. – М.: Наука, 2008. – 368 с.

14. Крылов В.И., Бобков В.В., Монастырский П.И. Вычислительные методы. М.: Наука, 1976.

15. Сборник задач по методам вычислений. Учебное пособие / Под ред. П.И.Монастырного. – 2-е изд. – Мн.: Университецкое, 2000. – 311 с.