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NEWTON'S METHOD OF SOLVING A SYSTEM OF NONLINEAR EQUATIONS
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Abstract. The concept of a system of nonlinear equations, the stages of solving the problem, the geometric interpretation of the solution of the equation and the concept of iterative processes are given and their application is shown in the examples. The problem of numerical solution of a number of practical problems consisting of a system of nonlinear equations is considered. There are a number of approximate computational methods for solving systems of nonlinear equations, including Newton's method. Using these methods, a number of specific practical problems were solved, a computational algorithm and a block diagram were developed. An approximate method of finding the true roots of a system of nonlinear equations is given, based on examples, graphs are used in the form of results, and appropriate conclusions are drawn.

Keywords: System of nonlinear equations, Maple, Newton's method, algorithm, graph, block diagram, approximate method, geometric interpretation, iterative process, numerical solution.

Many practical problems lead to the solution of a system of nonlinear equations. In general, a system of n unknown nonlinear algebraic or transcendental equations is written as follows:

$$\begin{cases} f(x_1, x_2, \dots, x_n) = 0 \\ f_3(x_1, x_2, \dots, x_n) = 0 \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots. \\ f_n(x_1, x_2, \dots, x_n) = 0 \end{cases} \quad (1)$$

This (1) system can be written in vector form as follows:

$$f(x) = 0. \quad (2)$$

here $x = (x_1, x_2, \dots, x_n)^T$ – vector column of arguments;

$(f_1, f_2, \dots, f_n)^T$ – vector column of functions; $(\dots)^T$ – sign of transponder operation. Search for a solution of a system of nonlinear equations – this is a much more complex problem than solving a single nonlinear equation. Generalizing the methods used to solve a single equation to solve a system of nonlinear equations requires a lot of calculations or cannot be applied in practice. In particular, this applies to the method of dividing the interval into two equal parts. Nevertheless, a number of iterative methods of solving nonlinear equations can be generalized to the solution of a system of nonlinear equations.

(2) We use the series approximation method to solve the system of equations. Suppose that vector (2) is isolated from equation

$x = (x_1, x_2, \dots, x_n)$ which is one of the roots k – approach

$x^k = \left(x_1^k, x_2^k, \dots x_n^k \right)$

be found. In this case (2) is the exact root of the vector equation

$x = x^k + \varepsilon^k$, (3)

can be expressed in the form, here $\varepsilon^k = \left(\varepsilon_1^k, \varepsilon_2^k, \dots \varepsilon_n^k \right)$ – error correction limit (root error).

Substituting (3) into (2), we obtain the following equation:

$f\left(x^{(k)} + \varepsilon^{(k)} \right) = 0.$ (4)

Suppose, $f (x)$ – this x and $x^{(k)}$ Let any bubble containing D be a continuously differentiable function in the field. (4) to the right of equation

side $\varepsilon^{(k)}$ - we spread the line by the levels of the small vectors and are limited to the linear terms of this line:

$f\left(x^{(k)} + \varepsilon^{(k)} \right) = f\left(x^{(k)} \right) + f'\left(x^{(k)} \right) \varepsilon^{(k)} = 0$ (5)

It follows from formula (5) that $f'(x)$ as a product $x_1, x_2, \dots, x_n$- in relation to variables $f_1, f_2, \dots, f_n$ The following Jacob matrix of the system of functions is understood:

$$f'(x) = W (x) = \begin{vmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_1} & \dots & \dfrac{\partial f_1}{\partial x_n} \\ \dfrac{\partial f_1}{\partial x_2} & \dfrac{\partial f_1}{\partial x_2} & \dots & \dfrac{\partial f_1}{\partial x_n} \\ \dots \dots & \dots \dots & & \dots \dots \\ \dfrac{\partial f_n}{\partial x_1} & \dfrac{\partial f_n}{\partial x_2} & \dots & \dfrac{\partial f_n}{\partial x_n} \end{vmatrix},$$

or if we write it in the form of a short vector,

$f'(x) = W (x) = \left| \dfrac{\partial f_i}{\partial x_j} \right|$, i,j = 1,n.

(5) the system is the limit that corrects this error $\varepsilon_i^{(k)}$ (i = 1,n)) is a linear system with matrix W (x). From this formula (5) can be written as follows:

$f\left(x^{(k)} \right) + W\left(x^{(k)} \right) \varepsilon^{(k)} = 0.$

From here, $W\left(x^{(k)} \right)$ - assuming a non-specific matrix, we have:

$\varepsilon^{(k)} = -W^{-1}\left(x^{(k)} \right) f\left(x^{(k)} \right)$

The result is this

$x^{(k+1)} = x^{(k)} - W^{-1}\left(x^{(k)} \right) f\left(x^{(k)} \right)$, k = 0,1,2, … (6)

We come to the formula of the Newtonian method, in this $x^{(0)}$ - as a zero approximation can be obtained the rough value of the root sought.

In practice (2), the calculations for solving a system of nonlinear equations by this method are continued according to formula (6) until the following condition is satisfied:

$|x^{(k+1)} + x^{(k+1)}|_\infty < \varepsilon . (7)$

Based on the above, Newton
We write the algorithm of the method as follows:
1. $x^{(0)}$ - initial approximation determined.
2. The value of the root is given by formula
 (6) determined by
3. If condition (7) is met, then
 the issue will be resolved and

$x^{(k+1)}$ – (2) of the vector equation
is taken as the root, otherwise
and go to step 2.In calculations
(2) of a system of nonlinear equations
 f (x) functions and their derivatives
the matrix W (x) is clearly given,
in which case it is a block diagram of the solution
of the system As shown in Figure 1.
 Figure 1. System of nonlinear equations
 algorithm of Newton's method for solving $f(x)$ the vector-function x is continuously differentiable up to twice around the root and the Jacob matrix $W(x)$ the non-specific, multidimensional Newtonian method has a quadratic approximation:
 $|x^{(k+1)} - x| < C |x^{(k)} - x|^2$
 We emphasize that the successful selection of the initial approximation is important to ensure the approximation of the method. As the number of equations increases and their complexity increases, the area of convergence narrows.

Special case. In computational practice, n = 2 is the most common. Do this, for example $f(z) = 0$ can also be seen in finding the complex roots of a nonlinear equation. Indeed, if this
 $f_1(x,y) = Re (f (x + jy)) $ and $f_2(x,y) = Im (f (x + jy))$
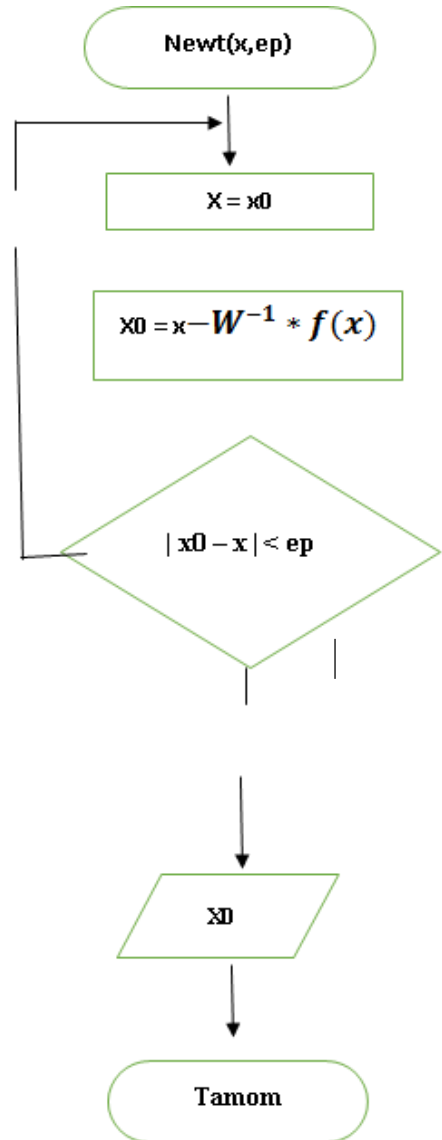
If we introduce the functions, z is the real part of the complex root x and the abstract part y is the approximate solution of the following two unknown systems of two nonlinear equations:

$$\begin{cases} f_1(x,y) = 0; \\ f_2(x,y) = 0, \end{cases} (8)$$

using Newton's method to calculate this approximation ε let's do it with precision.

D belongs to the field $X_0(x_0 y_0)$ - we choose the zero approximation. From (5) we can construct the following system of linear algebraic equations:

$$\frac{\partial f_1}{\partial x}(x - x_0) + \frac{\partial f_1}{\partial y}(y - y_0) = -f_1(x_0, y_0)$$

$$\frac{\partial f_2}{\partial x}(x - x_0) + \frac{\partial f_2}{\partial y}(y - y_0) = -f_2(x_0, y_0) \ (9)$$

We enter the following definitions:

$$x - x_0 = \Delta x_0, y - y_0 = \Delta y_0 \ (10)$$

(9) system Δx_0, Δy_0 for example, using the Kramer method. We write Kramer's formulas as follows:

$$\Delta x_0 = \frac{\Delta_1}{J}, \Delta y_0 = \frac{\Delta_2}{J}, (11)$$

where (9) the main determinant of the system is:

$$J = \begin{vmatrix} \frac{\partial f_1(x_0,y_0)}{\partial x} & \frac{\partial f_1(x_0,y_0)}{\partial y} \\ \frac{\partial f_2(x_0,y_0)}{\partial x} & \frac{\partial f_2(x_0,y_0)}{\partial y} \end{vmatrix} \neq 0, (12)$$

(9) The auxiliary determinants of the system are as follows:

$$\Delta_1 = \begin{vmatrix} -f_1(x_0,y_0) & \frac{\partial f_1(x_0,y_0)}{\partial y} \\ -f_2(x_0,y_0) & \frac{\partial f_2(x_0,y_0)}{\partial y} \end{vmatrix}; \Delta_2 = \begin{vmatrix} \frac{\partial f_1(x_0,y_0)}{\partial x} & -f_1(x_0,y_0) \\ \frac{\partial f_1(x_0,y_0)}{\partial x} & -f_2(x_0,y_0) \end{vmatrix}.$$

Δx_0, Δy_0 Substituting the found values of (10) into (9) of the system $X_1 = (x_1, y_1)$- find the components of the first approximation:

$$x_1 = x_0 + \Delta x_0, y_1 = y_0 + \Delta y_0 . (13)$$

We check the fulfillment of the following condition:

$$\boldsymbol{max}(|\Delta x_0|, |\Delta y_0|) \leq \varepsilon \ (14)$$

if this condition is met, then $X_1 = (x_1, y_1)$ we stop considering the first approximation (9) as an approximate solution of the system. If condition (14) is not met, then $x_0 = x_1$, $y_0 = y_1$ we construct a new system of linear algebraic equations (9). Take it off, $X_2 = (x_2, y_2)$- we find a tumor near the second. We check the solution with respect to (14). If this condition is met, then (9) is an approximate solution of the system $X_2 = (x_2, y_2)$ we accept. If condition (14) is not met, then $x_1 = x_2$, $y_1 = y_2$ that is $X_3 = (x_3, y_3)$ we

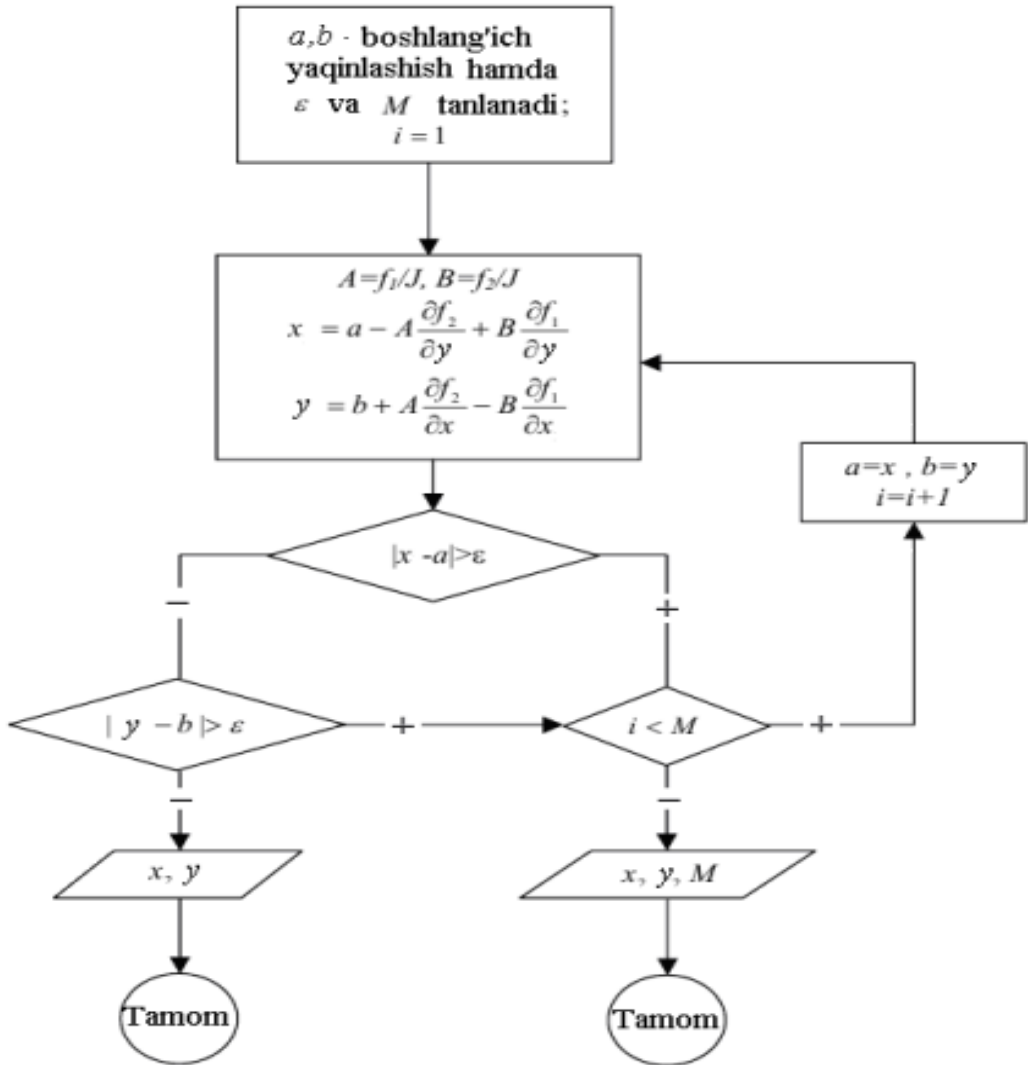create a new (1.8) system to find, and so on. A block diagram of solving this system is shown in Figure 2.

Figure 2. Newton's block diagram of the approximate solution of a system of two nonlinear equations with two unknowns.

1-for example. This

$$\begin{cases} f_1(x,y) = x^5 + y^3 - xy - 1 = 0; \\ f_1(x,y) = x^2y + y - 2 = 0. \end{cases}$$

the zero approximation of the system of equations $X_0 (x_0, y_0) = (2; 2)$ that is its exact solution $X = (x, y) = (1; 1)$ using Newton's method.

Solution: The process of solving an example, the iterations in iterations $X_k = (x_k, y_k)$ If you gain $\Delta X_k = (\Delta x_k, \Delta y_k)$ as in the following table:

k	x_k	y_k	$|X_k - X|$	$|\Delta X_k|/|\Delta X_{k-1}|^2$

0	2,000000000	2,000000000	1,414213562	-
1	1,693548387	0,890322581	0,702167004	0,351
2	1,394511613	0,750180529	0,466957365	0,947
3	1,192344147	0,82284086	0,261498732	1,199
4	1,077447418	0,918968807	0,112089950	1,639
5	1,022252471	0,976124950	0,032637256	0,032637256
6	1,002942200	0,996839728	4,317853366E - 3	4,054
7	1,000065121	0,999930102	9,553233627E-5	9,553233627E-5
8	1,000000033	0,999999964	0,999999964	5,337
9	1,000000000	1,000000000	1,272646866E-14	5,363

These results show that the iteration process is very fast - a solution of up to seven digits after the comma is obtained after eight iterations. A system of given equations

$$B = \begin{pmatrix} 0,032 & 0,0 \\ 0,0 & 0,9 \end{pmatrix}$$

if we solve it by the iteration method with the initial approximation, then the solution obtained with the comparative error is obtained after 247 iterations.

The numbers in the last column of the table confirm that the method has a quadratic approximation.

Indeed, $| \Delta X_k | \approx C | \Delta X_{k-1} |^2$ the connection is located close enough around the root that the constant C is large enough: $C \approx 5,4$. If the number of equations in the system of equations increases, then Jacob

we can see that the computational efficiency of Newton's method decreases due to the difficulty of calculating the matrix. If we look at the one-dimensional situation, there it is $f(x)$ and $f'(x)$ The difficulty of calculating is almost the same. In the N-dimensional case $f_i'(x)$ to calculate n^2 per calculation is required, which $f_i(x)$ means that it is several times more difficult to calculate n times.

2-Example. The following

$$\begin{cases} F(x, y) = 2x^3 - y^2 - 1 = 0 \\ G(x, y) = x y^3 - y - 4 = 0 \end{cases}$$

approximate the solution of the system in Newton's method.

Solution: Initial approximation graphically or selectively $x_0 = 1,2$ $y_0 = 1,7$ be identified. In that case

$$J(x_0, y_0) = \begin{vmatrix} 6x^2 - 2y \\ y^3 \ 3xy^2 - 1 \end{vmatrix}, \quad \text{demak} \quad J(1,2; 1,7) = \begin{vmatrix} 8,64 & -3,40 \\ 4,91 & 9,40 \end{vmatrix} =$$

97,910, Abbreviation to formula (13)

$$\begin{cases} x_1 = 1{,}2 - \dfrac{1}{97{,}91} \begin{vmatrix} -0{,}424 & -3{,}40 \\ 0{,}1956 & 9{,}40 \end{vmatrix} = 1{,}2 + 0{,}0349 = 1{,}2349 \\ y_1 = 1{,}7 - \dfrac{1}{97{,}91} \begin{vmatrix} 8{,}64 & -0{,}434 \\ 4{,}91 & 0{,}1956 \end{vmatrix} = 1{,}7 - 0{,}0390 = 1{,}6610 \end{cases}$$

Continuing the calculations in the same way,

$x_1 = 1{,}2343 \; y_1 = 1{,}6615$

In this example, we can see from the following Maple program and graphs that the system of equations has a single real solution (Figure 3):

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> plots[implicitplot]({2*x^3-y^2-1=0,x*y^3-y-4=0},x=-2..2,y=-3..3);
solve({2*x^3-y^2-1=0,x*y^3-y-4=0},{x,y});
allvalues(%);
evalf(%);
```

$\{\, x = 1.234274484,\ y = 1.661526467 \,\}$
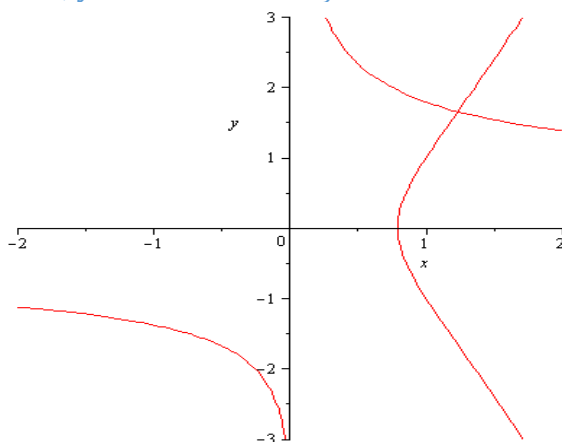
Figure 3. Graphs of functions in a system of equations given in the example drawn in Maple

Important results of his work are:

1. the solution of a system of nonlinear equations is much more complicated, and the problem is a perfectly unsolved problem of computational mathematics; 2. The initial problem of solving a system of nonlinear equations is the study of the existence, number and interval of solutions of a system of nonlinear equations, which are explained by solving specific examples;

3. the problem of finding the separated root of a system of nonlinear equations has been described in several approximate ways, explained by the solutions of concrete examples;

4. the approximate methods of finding the roots of a system of nonlinear equations have been studied from simple to complex and with their special cases, which has made it possible to shed more light on the subject;

5 The existence of real solutions of a system of equations, their number, the problem of finding the intervals in which these solutions lie, was studied

by drawing a graph of the functions of a system of nonlinear equations using the Maple package;

6. Newton's method is one of the most effective methods for solving a system of nonlinear equations, but its scope is very small;

7. The Newtonian method has a quadratic approximation rate;

Thus, the problem of solving a system of nonlinear equations depends on the type of practical problem, the choice of the correct approximate method and the initial condition, the effective use of these methods.

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TECHNOLOGY FOR CREATING A DEVICE FOR LAMINAR FLOW OF WATER IN PIPES
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Abstract: The article discusses the mathematical modeling of the movement of viscous incompressible fluids through a bundle of tubes located inside the outer pipe. The laminar and turbulent modes of this movement are considered, and the physical meaning of their occurrence is also analyzed. The fluid flow through n tubes of length L and radius r located inside the outer tube is considered. Calculation formulas are derived for calculating the maximum velocity of this flow, the volume of fluid passing through the cross section of the tube, the coefficient of resistance to friction in the tube along the length of the flow, and also the maximum value of the tangential stress. The results of the study of the relationship of the coefficient of resistance to friction in the tube with the Reynolds number are presented. A description is given of a device created according to the results of a study that brings the disordered flow of liquids into a laminar state.

Keywords: Reynolds number, laminar flow, turbulent flow, parabolic flow, friction force, integral, coordinate, pipe, viscosity, density, main flow velocity, average speed, maximum speed, radius, Hooke, Gegin, Poiseuille, Darcy-Weisbach, fluid volume, drag coefficient.

The motion of real fluids is often very different from that of laminar flow. They have a special property called turbulence. As the Reynolds number increases in real fluid flows in pipes, channels, and boundary layers, the transformation of a laminar-shaped flow into a turbulent flow is clearly observed. This transition of laminar flow to turbulent flow is sometimes called turbulence, which is fundamental in the whole field of hydrodynamics. Initially, such a transition was observed in the flow of straight pipes and channels.

Information on the forces acting on a fluid for flow in a cylindrical tube is given in the article [2; pp. 36-47].

Consider the motion of a tube of constant diameter along its entire length and the flow of fluid through n tubes of length L and radius r placed