NEW EXACT SOLUTIONS FOR THE LOADED KORTEWEG-DE VRIES AND THE LOADED MODIFIED KORTEWEG-DE VRIES BY THE FUNCTIONAL VARIABLE METHOD

Bazar Babajanov
Doctor of Sciences in Physical and Mathematical Sciences, Department of Applied Mathematics and Mathematical Physics, Urgench State University
Sadokat Maylieva
Master degree student, Urgench State University

Abstract: *In this paper, we construct exact traveling wave solutions of the loaded Korteweg-de Vries and the loaded modified Korteweg-de Vries by the functional variable method. The performance of this method is reliable and effective and gives the exact solitary and periodic wave solutions. All solutions to these equations have been examined and 3D graphics of the obtained solutions have been drawn using the MATLAB program. We get some traveling wave solutions, which are expressed by the hyperbolic functions and trigonometric functions. The graphical representations of some obtained solutions are demonstrated to better understand their physical features, including bell-shaped solitary wave solutions, singular soliton solutions, and solitary wave solutions of kink type. Our results reveal that the method is a very effective and straightforward way of formulating the exact traveling wave solutions of non-linear wave equations arising in mathematical physics and engineering.*

Key words: *the loaded Korteweg-de Vries equation, the loaded modified Korteweg-de Vries equation, periodic wave solutions, soliton wave solutions, functional variable method.*

The investigation of exact traveling wave solutions to non-linear evolution equations plays an important role in the study of non-linear physical phenomena. These equations arise in several fields of science, such as fluid dynamics, physics of plasmas, biological models, non-linear optics, chemical kinetics, quantum mechanics, ecological systems, electricity, ocean, and sea. One of the most important non-linear evolution equations is the Korteweg De Vries (KdV) equation.

The KdV equation was first observed by John Scott Russell in experiments, and then Lord Rayleigh and Joseph Boussinesq studied it theoretically. Finally, in 1895, Korteweg and De Vries formulated a model equation to describe the aforementioned water wave, which helped to prove the existence of solitary waves. In the mid-1960s, Zabusky and Kruskal discovered the remarkably stable particle-like behavior of solitary waves. The KdV equation is especially important due to the potential application of different properties of electrostatic waves in the development of new theories of chemical physics, space environments, plasma physics, fluid dynamics, astrophysics, optical physics, nuclear physics, geophysics, dusty plasma, fluid mechanics, and different other fields of applied physics [1, 2, 3, 4].

In recent years, studying electrostatic waves specifically to discuss different properties of solitary waves in the field of soliton dynamics has played a significant role for many researchers and has received considerable attention from them. The ion acoustic solitary wave is one of the fundamental non-linear wave phenomena appearing in plasma physics. In 1973, Hans Schamel studied a modified Korteweg-de Vries equation for ion-acoustic waves. The modified KdV equation has been applied widely in the molecular chain model, the generalized elastic solid, and so on [5, 6, 7]. Non-linear interactions between low-hybrid waves and plasmas can be described well by using the modified KdV equation [8].

In arterial mechanics, a model is widely used in which the artery is considered as a thin-walled prestressed elastic tube with a variable radius (or with stenosis), and blood as an ideal fluid [9]. The governing equation that models weakly nonlinear waves in such fluid-filled elastic tubes is the modified KdV equation

$$u_t - 6u^2 u_x + u_{xxx} - h(t)u_x = 0,$$

where t - is a scaled coordinate along the axis of the vessel after static deformation characterizing axisymmetric stenosis on the surface of the arterial wall. x - is a variable that depends on time and coordinates along the axis of the vessel. $h(t)$ - is a form of stenosis and characterizes the average axial velocity of the fluid.

We suppose that a form of stenosis $h(t)$ proportional to $u(0,t)$ and we consider the loaded KdV and the loaded modified KdV equation

$$u_t - 6\alpha u u_x + u_{xxx} + \gamma_1(t)u(0,t)u_x = 0,$$
$$u_t - 12\beta u^2 u_x + u_{xxx} + \gamma_2(t)u(0,t)u_x = 0,$$

where $u(x,t)$ is an unknown function, $x \in R$, $t \geq 0$, α and β are any constants, $\gamma_1(t)$ and $\gamma_2(t)$ are the given real continuous functions.

We establish exact traveling wave solutions of the loaded KdV and the loaded modified KdV by the functional variable method. The performance of this method is reliable and effective and gives the exact solitary wave solutions and periodic wave solutions. The traveling wave solutions obtained via this method are expressed by hyperbolic functions and trigonometric functions. The graphical representations of some obtained solutions are demonstrated to better understand their physical features, including bell-shaped solitary wave solutions, singular soliton solutions, and solitary wave solutions of kink type. This method presents wider applicability for handling non-linear wave equations.

References:

1. Sagdeev R.Z. (1966). Cooperative Phenomena and Shock Waves in Collision less Plasmas, Reviews of Plasma Physics, 4, 23-91.

2. Seadawy A.R., Cheemaa Nadia. (2020). Some new families of spiky solitary waves of one-dimensional higherorder KdV equation with power law nonlinearity in plasma physics, Indian Journal of Physics, 94(1), 117-126. https://doi.org/10.1007/s12648-019-01442-6.

3. Seadawy A.R., Cheemaa Nadia. (2019). Propagation of nonlinear complex waves for the coupled nonlinear Schr¨odinger Equations in two core optical fibers, Physica A: Statistical Mechanics and its Applications, 529(12), 13-30. https://doi.org/10.1016/j.physa.2019.121330.

4. Seadawy A.R., Cheemaa Nadia. (2019). Applications of extended modified auxiliary equation mapping method for high order dispersive extended nonlinear Schr¨odinger equation in nonlinear, Modern Physics Letters B, Volume 33(18), 1-11. https://doi.org/10.1142/S0217984919502038.

5. Gorbacheva, O.B. and Ostrovsky, L.A. (1983). Nonlinear vector waves in a mechanical model of a molecular chain, Physica D: Nonlinear Phenomena, 8(1-2), 223-228. https://doi.org/10.1016/0167-2789(83)90319-6.

6. Erbay, S. and Suhubi, E.S. (1989). Nonlinear wave propagation in micropolar media. II: Special cases, solitary waves and Painlev´e analysis, International Journal of Engineering Science, 27(8), 915-919. https://doi.org/10.1016/0020-7225(89)90032-3.

7. Zha, Q.L. and Li, Z.B. (2008). Darboux transformation and multi-solitons for complex mKdV equation, Chinese Physics Letters, Volume 25(1), 8. https://doi.org/10.1088/0256-307X/25/1/003.

8. Karney, C.F.F., Sen, A. and Chu, F.Y.F. (1979). Nonlinear evolution of lower hybrid waves, The Physics of Fluids, 22(5), 940-952. https://doi.org/10.1063/1.862688.

9. Demiray H. (2009). Variable coefficient modified KdV equation in fluid-filled elastic tubes with stenosis: Solitary waves, Chaos, Solitons and Fractals, 42, 358-364. https://doi.org/10.1016/j.chaos.2008.12.014.