SOLITON AND PERIODIC WAVE SOLUTIONS OF THE LOADED NONLINEAR EVOLUTION EQUATIONS

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Abstract: *In this article, we establish new traveling wave solutions for the loaded Benjamin-Bona-Mahony and the loaded modified Benjamin-Bona-Mahony equation by the functional variable method. The performance of this method is reliable and effective and gives the exact solitary wave solutions and periodic wave solutions. All solutions of these equations have been examined and three-dimensional graphics of the obtained solutions have been drawn by using the MATLAB program. We get some traveling wave solutions, which are expressed by the hyperbolic functions and trigonometric functions. This method is effective in finding exact solutions of many other similar equations.*

Key words: *loaded Benjamin-Bona-Mahony equation, loaded modified Benjamin-Bona-Mahony equation, hyperbolic functions, trigonometric functions, periodic wave solutions, solitary wave solutions, functional variable method.*

Benjamin-Bona-Mahony (BBM) equation is well known in the analysis of the surface waves of long wavelength in liquids, hydromagnetic waves in a cold plasma, acoustic-gravity waves in compressible fluids, and acoustic waves in harmonic crystals and it describes the model for propagation of long waves which incorporates nonlinear and dissipative effects [1]. In the last two decades, various versions of the BBM equation have been investigated in the literature [2].

In 1972, Benjamin, Bona, and Mahony formulated a model equation for the unidirectional propagation of small-amplitude long waves on the surface of water in a channel [3]. A general form of the BBM equation is

$$u_x + u_t - \alpha u u_x - u_{txx} = 0,$$

where $u(x,t)$ is an unknown function, $x \in R$, $t \geq 0$, α is any constant.

The BBM equation has been investigated as a regularized version of the KdV equation for shallow water waves [4]. In certain theoretical investigations the equation is studied as a model for long waves and from the standpoint of existence and stability, the equation offers considerable technical advantages over the KdV equation [5]. In addition to shallow water waves, the equation applies to the study of drift waves in plasma or the Rossby waves in rotating fluids. Under certain conditions, it also provides a model of one-dimensional transmitted waves.

The modified Benjamin-Bona-Mahony equation is a special type of the BBM equation. By changing the nonlinear term of the form $\alpha u^n u_x (n=2)$ the new modified form is obtained as follows:

$$u_x + u_t - \alpha u^2 u_x - u_{txx} = 0,$$

BBM equation can be solved by many methods. This equation is solved by (G'/G) - the expansion method [6], the exp-function method [7, 8], the homotopy perturbation method [9, 10], and the variation iteration method [11]. Zabusky and Kruskal investigated the interaction of solitary waves and the recurrence of initial states [12]. The Adomian decomposition method is another method to design some of the exact solitary wave solutions of the generalized form of the BBM equation [13]. Besides the analytical and exact solutions of the BBM equation, many numerical techniques from different families are developed and implemented for the numerical solutions to various evolution problems for the BBM equation [14, 15].

In this article, we consider the following the loaded BBM equation and the loaded modified BBM equation

$$u_x + u_t - \alpha u u_x - u_{txx} + \gamma_1(t) u(0,t) u_x = 0,$$
$$u_x + u_t - \beta u^2 u_x - u_{txx} + \gamma_2(t) u(0,t) u_x = 0,$$

where $u(x,t)$ is an unknown function, $x \in R$, $t \geq 0$, α and β are constants, $\gamma_1(t)$ and $\gamma_2(t)$ are the given real continuous functions.

We construct exact travelling wave solutions of the loaded BBM equation and modified BBM equation by the functional variable method. All solutions of these equations have been examined and three-dimensional graphics of the obtained solutions have been drawn by using the MATLAB program. We get some traveling wave solutions, which are expressed by the hyperbolic functions and trigonometric functions. The functional variable method is flexible, reliable and straightforward to find solutions of some nonlinear evolution equations arising in engineering and science.

References:

1. Abbasbandy S., Shirzadi A. The first integral method for modified Benjamin-Bona-Mahony equation, Communications in Nonlinear Science and Numerical Simulation, 2010, 15(7), 759-1764.

2. Hong B., Lu D. New exact solutions for the generalized BBM and Burgers-BBM equations, World Journal of Modelling and Simulation, 2008, 4, 243-249.

3. Benjamin T.B., Bona J.L., Mahony J.J. Model equations for long waves in nonlinear dispersive systems, Philosophical Transactions of the Royal Society London Series A, 1972, 27, 47-78.

4. Hereman W. Shallow water waves and solitary waves, Mathematics of Complexity and Dynamical Systems, 2011, 1520-1532.

5. Nickel J. Elliptic solutions to a generalized BBM equation, Physics Letters A, 2007, 364(3-4), 221-226.

6. Abazari R. Application of expansion method to travelling wave solutions of three nonlinear evolution equation, Computers and Fluids, 2010, 39(10), 1957-1963.

7. Bekir A., Boz A. Exact solutions for a class of nonlinear partial differential equations using exp-function method, International Journal of Nonlinear Sciences and Numerical Simulation, 2007, 8, 505-512.

8. He J.H., Wu X.H. Exp-function method for nonlinear wave equations, Chaos, Solitons and Fractals, 2006, 30, 700-708.

9. He J.H. Recent development of the homotopy perturbation method, Topological Methods in Nonlinear Analysis, 2008, 31, 205-209.

10. Sadighi A., Ganji D.D. Solution of the generalized nonlinear Boussinesq equation using homotopy perturbation and variational iteration methods, International Journal of Nonlinear Sciences and Numerical Simulation, 2007, 8(3), 435-443.

11. Wu X.H., He J.H. Solitary solutions, periodic solutions and compacton-like solutions using the exp-function method, Computers and Mathematics with Applications, 2007, 54(7-8), 966-986.

12. Zabusky N.J., Kruskal M.D. Interaction of solitons in a collisionless plasma and the recurrence of initial states, Physical Review Letters, 1965, 15(6), 240-243.

13. Kaya D., El-Sayed S.M. An application of the decomposition method for the generalized KdV and RLW equations. Chaos, Solitons and Fractals, 2003, 17, 869-877.

14. Dag I., Korkmaz A., Saka B. Cosine expansion-based differential quadrature algorithm for the numerical solution of the RLW equation, Numerical Methods for Partial Differential Equations, 2010, 26(3), 544-560.

15. Korkmaz A., Dag I. Numerical simulations of boundary-forced RLW equation with cubic b-spline-based differential quadrature methods, Arabian Journal for Science and Engineering, 2013, 38, 1151-1160.