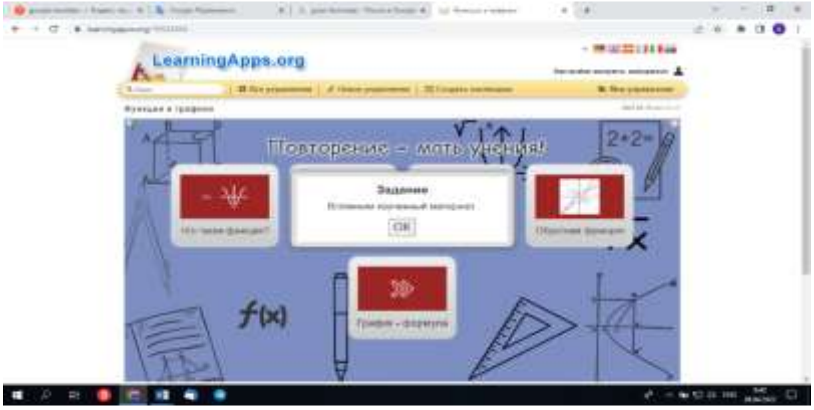
1-rasm. LearningApps xizmatida tayyorlangan videoma’ruzaga misol

Materialni taqdim etishning bunday formati talabalar tomonidan uni qanchalik yaxshi o'rganganligini darhol kuzatish va uni mustahkamlash uchun keyingi ishlarni sozlash imkonini beradi.

Shu bilan birga, shuni ta'kidlash kerakki, talabalarga videoma'ruza tomosha qilish jarayonida taqdim etiladigan topshiriqlar ham turli formatga ega bo'lishi mumkin: oddiy test topshiriqlari, boshqotirmalar ko'rinishidagi nostandart elementlar va hokazo (2-rasm).



2-rasm. Videoma’ruzaga topshiriq

Dars davomida, shuningdek, darsdan tashqari mashg‘ulotlarda o‘quvchilar bilimini yangilash va nazorat qilish bosqichlarini tashkil etish bo‘yicha ushbu onlayn xizmatning imkoniyatlari yanada qiziqroq. Buning sababi shundaki, unda nostandart (o'yin) shaklida javoblar tanlovi bilan vazifalarni yaratishga imkon beruvchi juda ko'p turli xil andozalar mavjud.

Shunday qilib, o'quv jarayonida interaktiv mashqlarni yaratish bo'yicha onlayn xizmatlardan foydalanish quyidagilarga imkon beradi: o'quv jarayonini o'quvchilarning shaxsiy xususiyatlari va ehtiyojlariga mos ravishda individuallashtirish; o'quv materialini o'quv faoliyatining turli usullarini hisobga olgan holda tashkil etish; vizual idrokni kuchaytirish va o'quv materialini o'zlashtirishni osonlashtirish; talabalarning kognitiv faolligini faollashtirish.

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SINGULYAR INTEGRAL UCHUN LOKAL BAHOLASH

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Annotatsiya: *Ushbu maqolada singulyar integralga trginometrik ko’phadlar orqaliyaqinlashishini zichlikning lokal uzluksizlik moduli oqali baholangan.*

Kalit so‘zlar: *Singulyar integral, funksiya uzluksizlik moduli, funksiyaning lokal uzluksizlik moduli, nuqtaning η – atrofi, eng yaxshi yaqinlashish.*

Quyidagi singulyar integralni qaraymiz

$$\bar{f}(x) = -\frac{1}{\pi}\int_{-\pi}^{\pi} f(\xi)\,\mathrm{ctg}\,\frac{\xi - x}{2}\,d\xi, \qquad (1)$$

bu erda $f \in C[-\pi,\pi]$.

Ma’lumki,

$$\int_0 \frac{\omega_f(\xi)}{\xi}\,d\xi < +\infty$$

shart bajarilganda $\bar{f}(x)$ funksiya uzluksiz va har bir nuqtada integral Koshining bosh qiymati ma’nosida mavjud, bu erda

$$\omega_f(\delta) = \sup_{|x_1 - x_2| < \delta} |f(x_1) - f(x_2)|, x_1, x_2 \in [-\pi,\pi]$$

funksiya f funksiyaning uzluksizlik moduli.

$[-\pi,\pi]$ kesmada berilgan x_0 nuqtaning $\eta(\eta > 0)$ atrofida berilgan $f(x)$ funksiyaning uzluksizlik modulini quyidagicha aniqlaymiz:

$$\omega_f^{x_0}(\delta,\eta) = \sup_{|x_1 - x_2| < \delta} |f(x_1) - f(x_2)|, x_1, x_2 \in [x_0 - \eta, x_0 + \eta], \delta < 2\eta.$$

$\omega_f^{x_0}(\delta,\eta)$ funksiya lokal uzluksizlik modulini deyiladi.

Osonlik bilan ko’rish mumkinki $\omega_f^{x_0}(\delta,\eta) \leq \omega_f(\delta)$.

Quyidagi belgilashni kiritamiz:

$$E_n(f) = \inf_{T_n} |f(x) - T_n(x)| \qquad (2)$$

Agar shunday $T_n(x)$ trigonometrik ko’phad mavjud bo’lib,

$$\lim_{n\to\infty} E_n(f) = 0$$

munosobat o’rinli bo’lsa, u holda $E_n(f)$ miqdorga $f(x)$ funksiyaga $T_n(x)$ trigonometrik ko’phad bo’yicha eng yaxshi yaqinlashish deyiladi[1].

$[-\pi,\pi]$ kesmada tayin x_0 nuqtaning $\eta(\eta > 0)$ atrofida berilgan $f(x)$ funksiyaning eng yaxshi yaqinlashish ham xuddi shunga o’xshash kiritiladi:

$$E_n^{x_0}(f) = \inf_{T_n} |f(x) - T_n(x)|, x \in [x_0 - \eta, x_0 + \eta], \delta < 2\eta$$

Bu ishda trigonometrik ko’phad qanday shartni qanoatlantirganda $E_n^{x_0}(\bar{f})$ lokal eng yaxshi yaqinlashishning nolga intilish tezligi singulyar integralning $\omega_{\bar{f}}^{x_0}(\delta,\eta)$ lokal uzluksizlik modulinining nolga intilish tezligi bilan bir xil bo’ladi. Bu savolga quyidagicha javob berish mumkin.

Faraz qilaylik, $h > 0$ va $t + h, t \in [x_0 - \eta, x_0 + \eta]$ bo’lsin. N nomerni shunday tanlaymizki $\frac{1}{2^{N+1}} < h < \frac{1}{2^N}$ tengsizlik o’rinli bo’lsin. U holda

$$|\bar f(t+h)-\bar f(t)|=\Big|\bar f(t+h)-T_{2^N}(t+h)+\sum_{j=1}^{N}\Big(T_{2^j}(t+h)-T_{2^{j-1}}(t+h)\Big)+$$

$$+T_0(t+h)-\left(\bar f(t)-T_{2^N}(t)+\sum_{j=1}^{N}\Big(T_{2^j}(t)-T_{2^{j-1}}(t)\Big)+T_0(t)\right)\Bigg|\le$$

$$\le\left|\bar f(t+h)-T_{2^N}(t+h)\right|+\left|\bar f(t)-T_{2^N}(t)\right|+|T_0(t+h)-T_0(t)|+$$

$$+\left|\sum_{j=1}^{N}\Big(T_{2^j}(t+h)-T_{2^j}(t)\Big)-\sum_{j=1}^{N}\Big(T_{2^{j-1}}(t+h)-T_{2^{j-1}}(t)\Big)\right|=J_1+J_2+J_3+J_4$$

J_1, J_2, J_3 larni baholash uchun

$$\left|\bar f(x)-T_n(\bar f,x)\right|\le$$

$$\le\frac{C}{2\pi}\ln h\cdot\omega_f^{x_0}(h,\eta)+\frac{C\pi^3 3^4}{2^3\cdot n^2}\cdot\omega_f^{x_0}(h,\eta)\cdot\left(lnh+\frac{1}{2}\right)+\frac{9C\pi}{4\pi}\cdot h\int_{\eta}^{\pi}\frac{\omega_f^{x_0}(y,y)}{y^2}dy.$$

tengsizligidan foydalanamiz. Endi J_4 ni baholaymiz

$$J_4=\left|\sum_{j=1}^{N}\Big(T_{2^j}(t+h)-T_{2^j}(t)\Big)-\sum_{j=1}^{N}\Big(T_{2^{j-1}}(t+h)-T_{2^{j-1}}(t)\Big)\right|=$$

$$=\left|\sum_{j=1}^{N}\int_0^h T'_{2^j}(t+u)du-\sum_{j=1}^{N}\int_0^h T'_{2^{j-1}}(t+u)du\right|=$$

$$=\left|\sum_{j=1}^{N}\int_0^h\Big(T_{2^j}(t+u)-T_{2^{j-1}}(t+u)\Big)'du\right|.$$

Endi quyidagi tengsizlikdan foydalayamiz:

Agar $T_n(x)$ - tartibi n dan katta bo'lmagan trigonometrik ko'phad bo'lib va ixtiyoriy $x\in O_\eta(x_0)$ uchun

$$|T_n(x)|\le M$$

tengsizligi bajarilsa, u holsa $x\in O_\eta(x_0)$ uchun

$$|T'_n(x)|\le M\cdot\frac{n}{\eta}$$

tengsizligi o'rinli.

Bu tsadiq Bernshteyin tengsisligining lokal analogi deyiladi va undan foydalansak, u holda quyidagi tengsislikni hosil qilamiz:

$$\left|T_{2^j}(t+u)-T_{2^{j-1}}(t+u)\right|\le\left|T_{2^j}(t+u)-f(t+u)\right|+$$

$$+\left|f(t+u)-T_{2^{j-1}}(t+u)\right|\le 2^j\omega_f^{x_0}\left(\frac{1}{2^j},\frac{1}{2^j}\right).$$

bo'lgani uchun

$$\left|\Big(T_{2^j}(t+u)-T_{2^{j-1}}(t+u)\Big)'\right|\le\left|T'_{2^j}(t+u)\right|+\left|T'_{2^{j-1}}(t+u)\right|\le$$

$$\le 2^j\omega_f^{x_0}\left(\frac{1}{2^j},\frac{1}{2^j}\right)\frac{\frac{1}{2^j}}{\eta}$$

bu erda $h=\frac{1}{2^j}$ va $h<2\eta$ lardan foydalanilsa quyidagi tengsizlikni olamiz

$$\left|\left(T_{2^j}(t+u)-T_{2^{j-1}}(t+u)\right)'\right| \le 2^j\,\omega_f^{x_0}\left(\frac{1}{2^j},\frac{1}{2^j}\right).$$

Shuning uchun,

$$|\bar f(t+h)-\bar f(t)| \le$$

$$\le \frac{C}{2\pi}\ln h\cdot\omega_f^{x_0}(h,\eta)+\frac{C\pi^3 3^4}{2^3\cdot n^2}\cdot\omega_f^{x_0}(h,\eta)\cdot\left(lnh+\frac{1}{2}\right)+\frac{9C\pi}{4\pi}\cdot h\int_\eta^\pi\frac{\omega_f^{x_0}(y,y)}{y^2}dy+$$

$$+\sum_{j=1}^N\int_0^h 2^j\omega_f^{x_0}\left(\frac{1}{2^j},\frac{1}{2^j}\right)du=\frac{C}{2\pi}\ln h\cdot\omega_f^{x_0}(h,\eta)+\frac{C\pi^3 3^4}{2^3\cdot n^2}\cdot\omega_f^{x_0}(h,\eta)\cdot\left(lnh+\frac{1}{2}\right)+$$

$$\frac{9C\pi}{4\pi}\cdot h\int_\eta^\pi\frac{\omega_f^{x_0}(y,y)}{y^2}dy+h\sum_{j=1}^N 2^j\omega_f^{x_0}\left(\frac{1}{2^j},\frac{1}{2^j}\right).$$

Agar

$$2^j\sim\int_{\frac{1}{2^j}}^{\frac{1}{2^{j-1}}}\frac{dy}{y^2}$$

ekanliligini hisobga olsak, u holda

$$h\sum_{j=1}^N 2^j\omega_f^{x_0}\left(\frac{1}{2^j},\frac{1}{2^j}\right)=h\sum_{j=1}^N\omega_f^{x_0}\left(\frac{1}{2^j},\frac{1}{2^j}\right)\int_{\frac{1}{2^j}}^{\frac{1}{2^{j-1}}}\frac{dy}{y^2}\le h\sum_{j=1}^N\int_{\frac{1}{2^j}}^{\frac{1}{2^{j-1}}}\frac{\omega_f^{x_0}(y,y)}{y^2}dy$$

$$\le h\int_h^\pi\frac{\omega_f^{x_0}(y,y)}{y^2}dy.$$

Shunday qilib,

$$|\bar f(t+h)-\bar f(t)|\le const\left[\left(lnh+h^2+\frac{1}{2}\right)\omega_f^{x_0}(h,\eta)+h\int_\eta^\pi\frac{\omega_f^{x_0}(y,y)}{y^2}dy\right].$$

Teorema. Agar $h>0, t+h, t\in[x_0-\eta, x_0+\eta]$ bo'lib, $\bar f(x)\in C[-\pi,\pi]$ uchun

$$|\bar f(x)-T_n(\bar f,x)|\le const\left[ln\frac{1}{n}\cdot\omega_f^{x_0}\left(\frac{1}{n},\eta\right)+\frac{1}{n}\int_\eta^\pi\frac{\omega_f^{x_0}(y,y)}{y^2}dy\right]$$

tengsizlik o'rinli bo'lsa, u holda quyidagi tengsizlik o'rinli bo'ladi[3].

$$|\bar f(t+h)-\bar f(t)|\le const\left[\left(lnh+h^2+\frac{1}{2}\right)\omega_f^{x_0}(h,\eta)+h\int_\eta^\pi\frac{\omega_f^{x_0}(y,y)}{y^2}dy\right].$$

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